LMS I/Q imbalance correction scheme for I/Q receivers

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Block Diagram

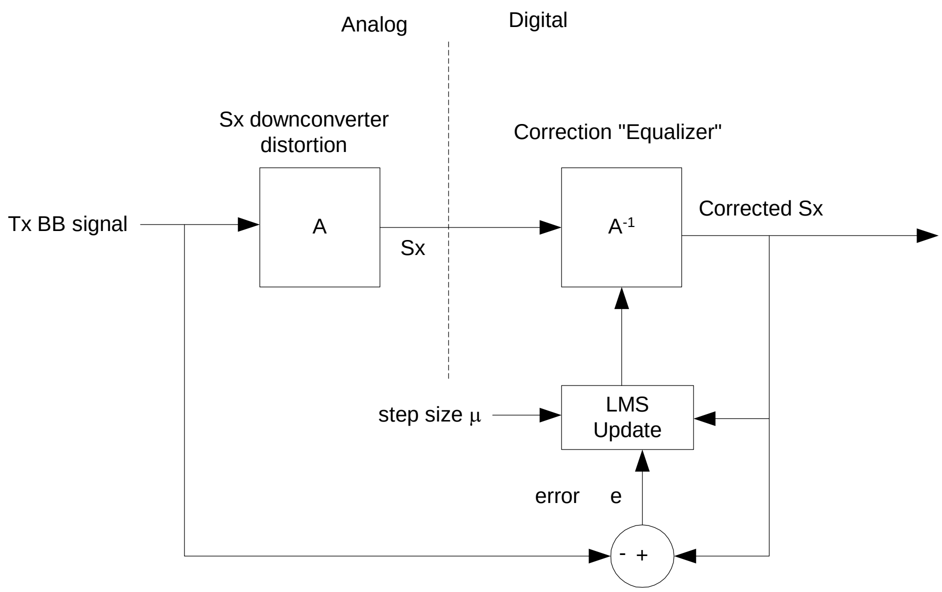

Figure 1. **Top level block diagram.**

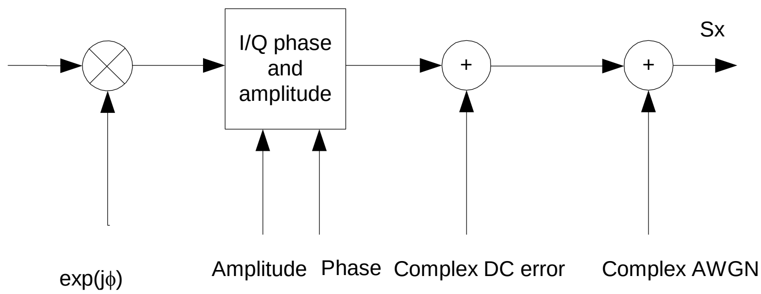

Figure 2. **Block A distortion generator showing details.**

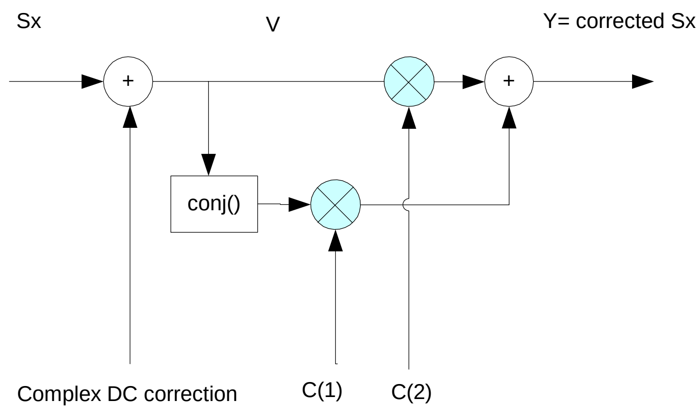

Figure 3. Correction block details.

Description

Distortion generation

The signal can be distorted by doing the following operations;

- Multiply the magnitudes of the I and Q vectors by different values close to one to achieve an amplitude difference between I and Q.
- Rotate the Q vector relative to the I vector such that the phase difference between them is no longer 90 degrees.
- Rotate both the I and Q by a fixed phase difference to simulate a group delay induced phase rotation.
- Add an independent DC offsets to both I and Q inputs.
- Add an independent noise to both I and Q inputs.

The ideal transmitted signal T can be represented by the following formula;

$$T = i + jq$$ **Equation 1**

Where

- i is the in-phase component of the ideal signal T
- q is the quadrature component of the ideal signal T
- j is the imaginary operator

The measured Sx signal experiences distortion and can be represented by the following formula;

$$S = e^{j\Omega}(\alpha i + j\beta q \bullet e^{j\varphi}) + d + n$$ **Equation 2**

Where

- S is the distorted signal measured at the Sx ADC,

- Ω is a common rotation applied to both *i* and *q* components to simulate group delay from the Tx to Sx.
- φ is the rotation applied to the *q* component only, and represents the phase difference from ideal (i.e. when *i* and *q* are 90 degrees out of phase with each other)
- α and β are independent gains applied to the I and Q paths
- *d* is a complex dc offset
- *n* is complex noise with variance σ^2

The distortion matrix A in Figure 1 represents the overall transformation.

Distortion correction

The distortion correction is done by applying an estimate of the inverse A^{-1}. Hence, this is actually a form of equalization. There is a mathematical equivalence between a complex representation of I/Q distortion and a more conventional form using sine and cosine leakages as in [1]. Essentially this complex technique is analogous to adding differential gain and phase imbalances. Refer to reference [2] for more details.

The complex representation of the equalizer is assumed here and is shown in Figure 3.

The equalizer operation is formulated as follows;

$$V = \begin{bmatrix} (S+\hat{d}) \\ (S+\hat{d})^* \end{bmatrix}$$ **Equation 3**

Where

- S is the incoming (distorted) signal which is complex
- $(\ )^*$ is the complex conjugate operator
- $\hat{d}$ is the negative estimate of the complex DC offset, i.e. $\hat{d} \approx -d$

The output from the equalizer is computed as

$$Y = \hat{C}'V$$ **Equation 4**

where $\hat{C}$ is a complex vector represented by two complex values $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ which are the complex tap weights for the equalizer.

$$\hat{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$ **Equation 5**

Algorithm

We attempt to get back the original signal by estimating the inverse A^{-1}. Estimation involves computing estimates for ***C***, and *d* using the LMS update algorithm.

Initialization

Set $\hat{C} = \begin{bmatrix} 0 + j*0 \\ 0 + j*0 \end{bmatrix}$

Set $\hat{d} = 0 + j*0$

Set $\mu = 0.1$ or some other small value.

Recursion

- Take a complex sample of the Tx signal (*T(n)*) and the Sx signal (*S(n)*) at time n.
- Compute the output from the equalizer, i.e.

$$V = \begin{bmatrix} (S + \hat{d}) \\ (S + \hat{d})* \end{bmatrix}$$ **Equation 6**

$$Y = \hat{C}'V$$ **Equation 7**

- Compute the complex error

$$E(n) = Y(n) - T(n)$$ **Equation 8**

- Apply the complex updates

$$\hat{C}(n+1) = \hat{C}(n) - \mu V*(n)E(n)$$ **Equation 9**

$$\hat{d}(n+1) = \hat{d}(n) - \mu E(n)$$ **Equation 10**

- Go back to the beginning

In order to converge, the DC errors must also be minimized. Since DC compensation already occurs in the Sx path as part of the APD algorithm , ***equation 10*** does NOT need to be implemented.

Simulation Results

For the simulations, a common α and d were applied to illustrate the convergence.

Parameters

Step size=0.2 initially, change to 0.02 after 160 iterations (tracking mode).
I/Q amplitude error = -39%
I/Q phase error =-30 degrees
Group delay phase shift = 15 degrees
DC offset d=0.1-j0.2
AWGN noise variance = 0.02 (SNR approx 33dB SNR).

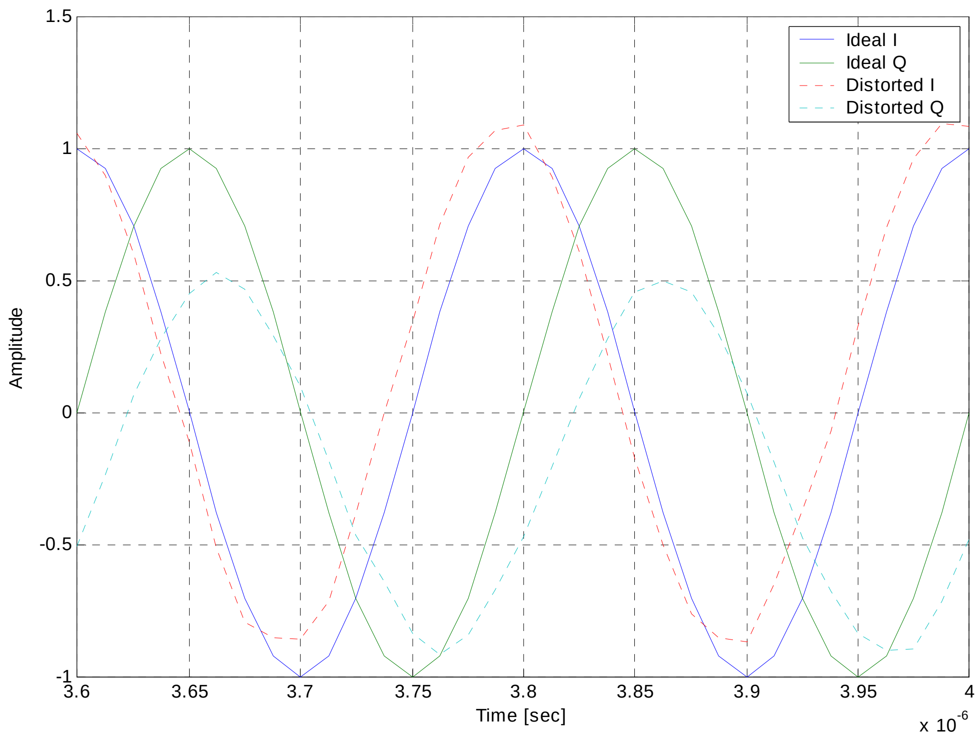

Figure 4. Sx path with -39 % I/Q amplitude error, -30 degrees I/Q phase error, a 15 degree group phase error, noise and a 0.1-j0.2 dc offset added.

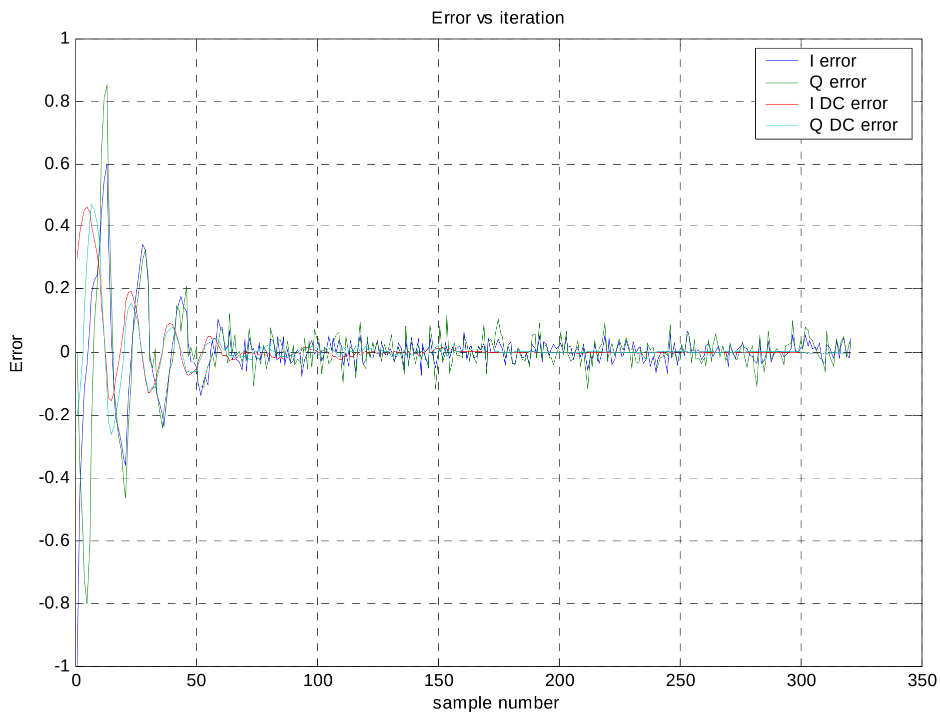

Figure 5 **Convergence of the complex error vs. sample number (assuming 80Mhz sampling rate). Initial step size =0.2, scaled to 0.02 after 160 iterations (tracking mode).**

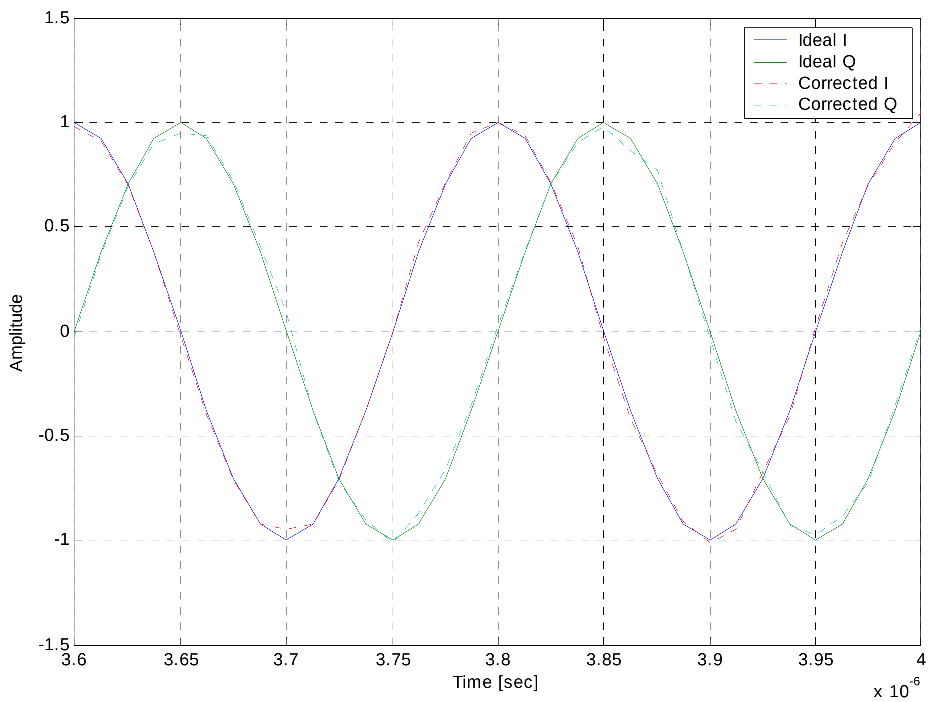

Figure 6 **Signal after convergence, better than 40 dB IIR.**

Advantages of the algorithm

- Although the simulation results are shown for a 5MHz tone, the algorithm can take ANY signal at its input, which means that we could apply this algorithm during the first part of the TX signal when the short symbols are being transmitted.
- An analog loop-back is not required.
- Alternatively, a set of training tones could be appended to the end of the transmit packet.
- The training can be started and stopped (i.e. "held") by setting the step size μ=0.
- The training can be implemented all at once or "intermittently" for as long or as little time as necessary (strictly speaking, as little as one sample every *k* packets is possible)

References

[1] "Adaptive Compensation for Imbalance and Offset Losses in Direct Conversion Transceivers", James Cavers, Maria Liao, *IEEE Trans. On Veh. Tech., Vol 42, No. 4, November 1993*, pp. 581-588.
[2] "*Complex Mathematical Representation of I/Q imbalances*", Neil Birkett, Dec. 2004.