Complex Mathematical Representation of I/Q imbalances

Neil Birkett Jan 13, 2005

Introduction

I/Q imbalance can be represented as a leakage from the Q to I path [1]. One representation is to put the entire phase imbalance into the sine path.

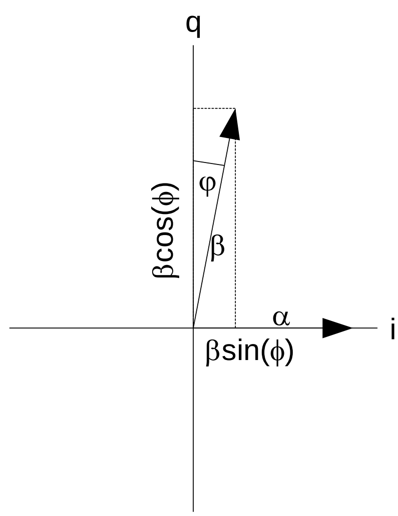

Figure 1 **Phasor representation of I/Q imbalance.**

In [1] this is represented as follows;

$$M = \begin{bmatrix} \alpha & \beta\sin(\phi) \\ 0 & \beta\cos(\phi) \end{bmatrix}$$ **Equation 1**

where α and β are measures of the gain imbalance and ϕ is the phase imbalance.

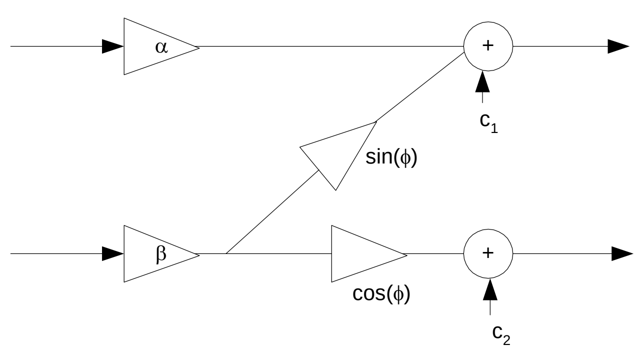

Figure 2. **Conventional implementation of IQ imbalance.**

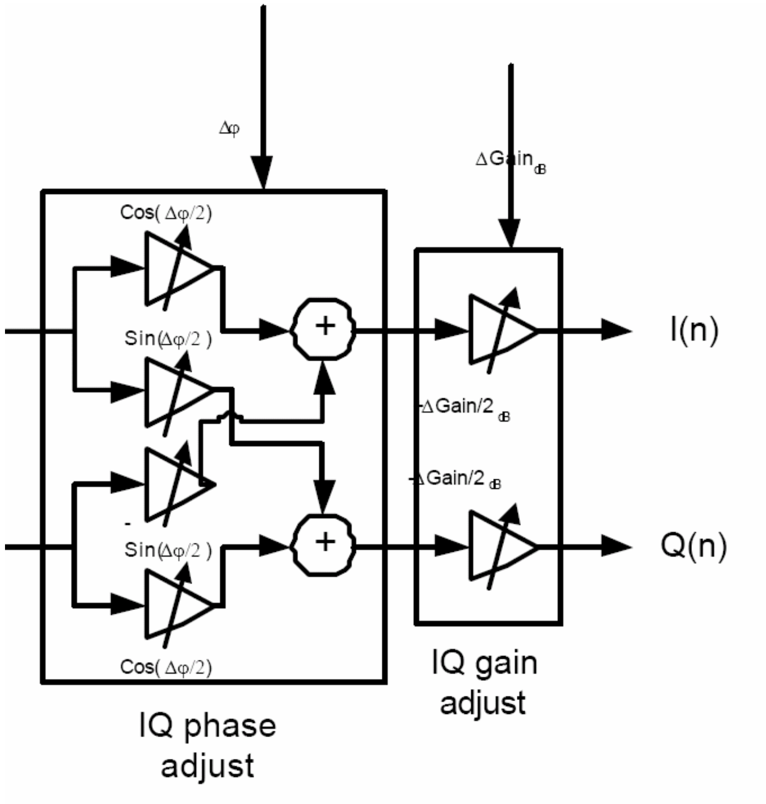

Figure 3 **Alternate implementation of previous figure has 6 adjustments to be made.**

An alternate representation of the distortion and can be stated as;

$$y = \alpha i + j\beta q \bullet e^{j\varphi} + d \qquad \textbf{Equation 2}$$

Where

- *y* is the complex distorted signal,
- φ is the rotation applied to the *q* component only, and represents the phase difference from ideal (i.e. when *i* and *q* are 90 degrees out of phase with each other),
- α and β are independent gains applied to the I and Q paths.
- *i* is the in-phase component of the ideal signal T
- *q* is the quadrature component of the ideal signal T
- *j* is the imaginary operator
- d is a complex dc offset.

Ignoring DC offset, we can define a normalized version as

$$y' = i + j\frac{\beta}{\alpha} q \bullet e^{j\varphi} = i + j\gamma q \qquad \textbf{Equation 3}$$

where $\gamma = \Delta e^{j\varphi}$ is a complex number and we have defined $\Delta = \dfrac{\beta}{\alpha}$ as a measure of the amplitude imbalance.

New Representation

It is possible to construct an alternate structure using complex taps to generate *y*.

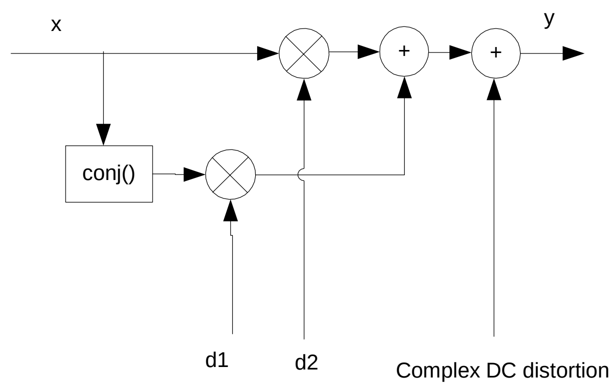

Figure 4. Alternate (Complex) implementation of I/Q imbalance.

The math behind the new representation

As per Figure 4, and ignoring the dc offset, let us define the output *y* as;

$$y = d_1 x + d_2 x^* \qquad \textbf{Equation 4}$$

where

- d_1 and d_2 represent the complex distortion multipliers
- * is the conjugate operator
- x is the complex input.

Similar to Equation 3, we can replace x with $i + jq$ and normalize the expression as follows;

$$y = d_1(i + jq) + d_2(i - jq) = i(d_1 + d_2) + jq(d_1 - d_2) \qquad \textbf{Equation 5}$$

$$y' = i + jq\left[\frac{d_1 - d_2}{d_1 + d_2}\right] \qquad \textbf{Equation 6}$$

We can equate Equation 3 and Equation 6 to obtain the equality below;

$$\left[\frac{d_1 - d_2}{d_1 + d_2}\right] = \gamma = \Delta e^{j\varphi} \qquad \textbf{Equation 7}$$

Lets examine the left side of equation 7 and rewrite it.

$$\left[\frac{d_1 - d_2}{d_1 + d_2}\right] = \left[\frac{1 - d_2/d_1}{1 + d_2/d_1}\right] = \left[\frac{1-z}{1+z}\right] \qquad \textbf{Equation 8}$$

where $z = d_2/d_1$

If we now let z equal some arbitrary complex number $a + jb$, we can substitute into equation 8 and simplify to get;

$$\left[\frac{1-z}{1+z}\right] = \frac{1 - a^2 - b^2 - 2jb}{1 + 2a + a^2 + b^2} \qquad \textbf{Equation 9}$$

We can rewrite equation 7 using Euler's formula and equating to Equation 9 to get,

$$\Delta e^{j\varphi} = \Delta\cos\varphi + j\Delta\sin\varphi = \frac{1 - a^2 - b^2 - 2jb}{1 + 2a + a^2 + b^2} \qquad \textbf{Equation 10}$$

Hence, from equation 10 we can generate two equations (real and imaginary) with two unknowns;

$$\begin{aligned} \Delta\cos\varphi &= \frac{1 - a^2 - b^2}{1 + 2a + a^2 + b^2} \\ \Delta\sin\varphi &= \frac{2b}{1 + 2a + a^2 + b^2} \end{aligned} \qquad \textbf{Equation 11}$$

It is now possible to solve for *a* and *b* for values of Δ and ϕ using equation 11.

References

[1] "Adaptive Compensation for Imbalance and Offset Losses in Direct Conversion Transceivers", James Cavers, Maria Liao, *IEEE Trans. On Veh. Tech., Vol 42, No. 4, November 1993*, pp. 581-588.

[2]